IFWG Memo: Telescope Primary Mirror Size

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Revision: 1.6 last modified on Date: 2003/03/25 18:07:13

Objective

This memo addresses a single question: what is the minimum diameter of the unit telescope primary mirrors needed to observe the faintest sources of high scientific interest, i.e. active galactic nuclei and quasars, with the MROI?

1 Summary

The appropriate metric to use in addressing this question is the limiting magnitude for self-referenced stabilisation of the fringes against the effects of atmospheric turbulence. The unit telescopes must collect sufficient photons to run both fringe-tracking and tip-tilt correction systems, for a reasonable sample of active galactic nuclei (AGN).

The inner parts of AGN have never been resolved, so a reasonable sample brighter than the limiting sensitivity is needed to ensure that more than a handful of these are compact enough to be imaged by MROI. A sensitivity of 14th magnitude at H band will be sufficient to permit observations of around 150 AGN.

Assuming moderately good seeing of 0.7 arcsec FWHM, we find that a minimum telescope diameter of 1.3 m is required to reach a limiting magnitude for fringe envelope tracking of 14 at H band, provided *all* the following goals are met:

- 20% throughput from the top of the atmosphere to detected photons
- 1.5e detector read noise for the fringe tracker
- Total spatial wavefront error $\sigma_{\text{spatial}} = \lambda/14$ over the full aperture at $\lambda = 1.65\,\mu$m
- Total temporal phase jitter $\sigma_{\text{temporal}} = \lambda/14$ in a 36ms integration at $\lambda = 1.65\,\mu$m
- A tip-tilt system that degrades the H-band visibility from the value for perfect tip-tilt correction by less than 10%
- The tip-tilt system is likely to use visible light — a typical magnitude at which the above performance goal must be met is $V = 16$

2 Context

This document attempts to:

1. Convert a critical science goal for MROI into a requirement for the limiting sensitivity of the interferometer.
2. Determine what unit telescope size is needed to meet this requirement.

In addressing (2), we present a signal-to-noise calculation for the interferometer. This calculation depends somewhat on the performance of sub-systems that have not yet been fully specified, so we take the approach of setting *goals* for the performance of these sub-systems. We only set such goals where these have an impact on the limiting sensitivity of the interferometer.

This memo is based on sections of the System Design document that were written by David Buscher. The calculations have been checked by JSY.

3 Sensitivity requirement

AGN are the only feasible extragalactic targets for the MRO Interferometer (MROI). Nearby examples are expected to have structure (the molecular torus and the outer parts of the broad-line region) on the angular scales accessible to MROI. Low-redshift AGN are bright enough to observe with MROI, provided care is taken with the interferometer design.

The extra-galactic component of the top-level science mission for the MROI sets the basic requirement for its desired sensitivity. At K-band magnitudes fainter than about 11 the very closest and brightest active galactic nuclei and quasars just start to become visible (henceforth we will use AGN as a generic term for active galactic nuclei and quasars). However, it is not until a K-band sensitivity of 13 is reached that of order 50 targets become visible in the Northern celestial sky. A magnitude limit of 14 at K would give approximately 250 potential targets.

The lower thermal background at $1.65\,\mu$m implies that the MROI will have its best fringe-tracking sensitivity in the H band. We have extracted numbers of quasars as a function of limiting V magnitude from the catalogue of Veron-Cetty & Veron (A&A, 374, 92). These data are shown in Figure 1 after conversion from V magnitude to H magnitude using $V-K=2.40$ and $H-K=0.75$ (Enya *et. al.* , ApJS, 141, 23).

We have chosen to extract numbers of *quasars* from the catalogue because these are the most core-dominated sources, and so magnitudes obtained in large (few arcsecond) apertures should be representative of the nuclear component, without significant contamination from the light of the host galaxy.

That said, the typical angular size of the molecular torus which supposedly dominates the near-IR nuclear emission is unknown. To guarantee some observable sources, it is

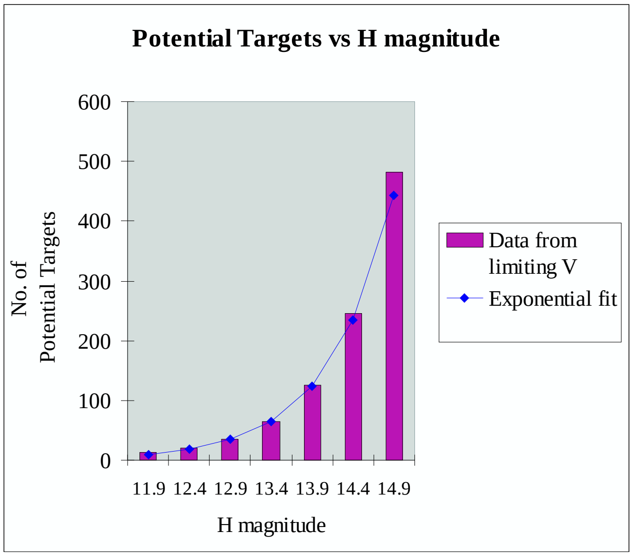

Figure 1: Histogram showing number of accessible quasars as a function of H-band sensitivity.

important to have a reasonable sample of AGN with fluxes above the limiting sensitivity. For this reason we adopt a limiting sensitivity of $H = 14$, giving access to around 150 potential targets.

Even if 80% of these AGN were found to be too large to observe, the $H = 14$ limit would permit the MROI to image a sample of 30. No competitor array will be able to image more than the brightest few AGN.

As will be expanded on later, the concept of sensitivity for an optical/infrared interferometer has a very precise meaning. In particular, what we mean by a "sensitivity" of 14th magnitude in a given band is that some form of fringe tracking — either phase or envelope tracking — be possible with a source of that brightness.

We have only considered using the science target itself as a reference, as the sky coverage will be of order 1% when using off-axis reference stars brighter than 14th magnitude (according to Bahcall & Soneira, ApJS, 44, 73, the density of stars brighter than $V = 14$ is typically a few hundred per square degree). In any case, we will not be able to afford dual-feed capability in Phase 1 of the MROI.

In practice tip-tilt correction is also needed to meet the fringe tracking requirement when using moderately-sized telescope apertures. It has been suggested that the tip tilt system might use some fraction of the $UBVRI$ light. If so, typical AGN colors suggest it must work at a V magnitude of 16.

In Section 4.2 we set a goal for the performance of the tip-tit system under these conditions. It was demonstrated in the System Design document that this goal can be met

— we leave it to Dick Horton and Gary Loos to revisit this point in their forthcoming memo on the specification for the tip-tilt system.

4 Meeting the requirement

4.1 The interferometric signal-to-noise ratio

We assume that the interferometer collects a sequence of "exposures" of fringe patterns, each with some integration time which we will refer to hereafter as the "coherent integration time". These data can then be averaged incoherently to achieve an increased SNR. The incoherent integration time can in principle be increased indefinitely by repeating the observation at the same sidereal time (to get the same projected baseline) on many nights in succession. Hence, there is no *fundamental* limit to the SNR of a visibility modulus or closure phase measurement as long as the source does not change over the timescale of the measurements.

However, the Earth's atmosphere perturbs the differential paths to the array elements by many tens of microns on timescales of a few seconds. If no attempt is made to follow these path fluctuations it is likely that no fringes will be seen for most exposures since the differential paths will be much greater than the coherence length of the light being measured.

Tracking these path fluctuations relies on being able to derive a path-error signal with a suitable SNR on a timescale less than the timescale on which the atmosphere moves the fringes out of view. So, if there are too few photons available to do the fringe tracking, or the fringes have too low a contrast, the source simply cannot be observed (at least in a baseline boot-strapping mode).

In the photon-noise-limited regime, the SNR of all fringe tracking schemes is essentially a monotonic function of $< V^2 > N$ where $< V^2 >$ is the mean-squared apparent fringe visibility and N is the mean number of photons received in a coherent integration time. Because the dependence on V is quadratic, more stringent requirements must be set on the apparent fringe contrast than on the photon throughput.

4.2 Wavefront error budget

Aberrations introduced by the Earth's atmosphere will reduce the fringe contrast by an amount which depends on the aperture size used and the coherent integration time. Any extra spatial aberrations and pathlength fluctuations introduced by the interferometer optics will further reduce V.

If the interferometer optics introduce random wavefront aberrations with an rms value of $\sigma_{\rm spatial}$ radians into the light from each telescope, the resulting fringes will have a visibility reduction of approximately $\exp(-\sigma^2_{\rm spatial})$. Similarly, if fluctuations in the internal paths inside the interferometer introduce an rms phase jitter of $\sigma_{\rm temporal}$ radians into each beam during a coherent integration time, the contrast will also be reduced by a factor of $\exp(-\sigma^2_{\mathrm{temporal}})$.

Hence, if the interferometer optics introduce spatial aberrations at the level of $\sigma_{\mathrm{spatial}} = \lambda/14$ and a temporal piston jitter of $\sigma_{\mathrm{temporal}} = \lambda/14$ then each of these two effects will introduce a reduction in the observed visibility by a factor 0.8. We adopt these as goals for the entire interferometer, with the exclusion of the fast tip-tilt system. For the latter we adopt the requirement that any residual errors in tilt-correction should lead to a visibility loss (on top of that for perfect tip-tilt correction) of no more than 10%.

For the purpose of meeting the sensitivity requirement, these goals for wavefront errors need only be met at H band.

4.3 Throughput

A goal for the interferometer is to have 20% throughput from the top of the Earth's atmosphere to detected photons. At the COAST array in Cambridge, the throughput has been measured to be $\sim 4.3\%$ at a wavelength where the detector quantum efficiency is 29%. Replacement with a Rockwell FPA with a quoted efficiency of 70% would thus give the COAST system a throughput of 10%. At COAST we have not seriously attempted to optimize the array throughput and so a goal of 20% for the MROI does not seem unreasonable.

4.4 Limiting magnitude for fringe tracking

We calculate here whether the proposed interferometer will be able to observe the faint sources which are of scientific interest. We will take as a test case the observation of an AGN with an unresolved core magnitude of $H = 14$, and $V = 16$ in the aperture used by the tip-tilt system. Whether fringe data can be taken on a faint source will be set by two factors. Firstly if the fringe tracker does not have enough light to track fringes then the source cannot be observed. Secondly, if the tip-tilt system does not have a sufficiently high SNR in the wavefront sensor to adequately correct the wavefront then the fringe contrast will be reduced and so the fringe-tracker SNR may be reduced below a level where fringe tracking is possible.

4.4.1 Assumptions

For the purposes of this section we assume an interferometer with 1.3 meter diameter apertures with tip-tilt correction only. We assume that the interferometer achieves the stated design goals so that the system throughput from the sky to detected photons is 20% and that the degradation in fringe contrast due to spatial wavefront errors in the interferometer is 0.8, that the degradation due to temporal phase jitter is 0.8 and that the degradation due to imperfect tip-tilt tracking is 0.9, leading to a total degradation by a factor of 0.57. We further assume that the interferometer operates a group-delay fringe tracker which tracks the fringe envelope using all of the H-band light with a center wavelength of $1.65\,\mu$m and a bandpass FWHM of $0.3\,\mu$m. The fringe tracker uses a pairwise combiner with 5 spectral channels and a detector with read noise $\sigma = 1.5$ electrons. The seeing is assumed to be $r_0 = 14\,$cm and $t_0 = 4.3\,$ms measured at $\lambda = 500\,$nm.

4.4.2 Fringe-tracking SNR

For a set of pair-wise beam-combiners, each output of the combiner receives the equivalent of half of the light from one telescope (the light from N telescopes appears at $2N$ outputs). When observing an object with a H magnitude of 14 then 34 photons are detected during each integration time (assumed to be $2t_0$ at $\lambda = 1.65\mu$m, i.e. 36 ms) at each output. The fringe detection scheme is assumed to be one where 2 intensity measurements are made at each output with $\pi/2$ phase steps introduced between them. The two outputs of the beam-combiner are π out of phase so the end result is a set of 4 measurements equally spread across 0 to 2π in phase. Thus there are 20 pixel reads to detect the photons across the 5 spectral channels and 68 photons from the source detected in total.

The sky background in the H band as measured in La Palma is equivalent to 14.7 magnitudes/arcsec^2. We would expect the higher-altitude MRO site to have no worse sky background, so for a 0.5×0.5 arcsec aperture, we would expect to detect at most 9 photons from the sky background over the spectral band. In the H band we expect less than 1 photon per integration from the thermal emissivity of the interferometer optics, assuming 300K optics with a total emissivity of 80%.

The rms fringe contrast degradation for a perfectly tip-tilt corrected aperture of diameter $2.2r_0$ is 0.61 and the reduction due to a $2t_0$ integration is 0.79. Combining this with the factor 0.57 reduction due to system imperfections gives an rms fringe contrast reduction of 0.275. The formulae employed for the SNR calculations below use a definition of the fringe visibility V in terms of the normalized Fourier amplitude. For a unit-contrast fringe $V = 0.5$, so the measured value of V is 0.138 when observing an unresolved source.

The ability to track the group delay envelope depends on a monotonic function of a fringe-tracking signal-to-noise ratio defined as

$$\mathrm{SNR}_{\mathrm{track}} = \frac{< V^2 > N^2}{N + N_{\mathrm{background}} + n_{\mathrm{pix}}\sigma^2}$$

where $< V^2 >$ is the mean squared fringe visibility, N is the number of photons detected from the source per coherent integration, $N_{\mathrm{background}}$ is the number of thermal and sky background photons detected in this time, n_{pix} is the number of pixel reads and σ is the per-pixel read noise. This formula is derived from Buscher 1988 (PhD thesis, University of Cambridge), section 5.1.2 but with the addition of the effects of read and background noise. For the observation of an $H = 14$ unresolved source this SNR has a value of 0.71.

Section 5.1.2 of Buscher 1988 presents simulations of fringe tracking with a 5-spectral-channel group delay system at different SNRs. It was found that fringe tracking could be achieved for values of $\mathrm{SNR}_{\mathrm{track}}$ down to 0.33 (N.B. this value has been converted from the "canonical SNR" used in that section). Thus envelope-tracking on a $H = 14$ source should be possible with more than a factor of two margin of safety.

Factors which would reduce this margin of safety include (a) source resolution (b) not meeting throughput and wavefront quality goals and (c) not meeting the detector read noise goal. Factors which could increase the margin of safety include (a) using more advanced fringe-tracking algorithms, particularly those involving detection of linear phase drifts (see Buscher 1988 section 5.1.1) (b) using spatial filtering to increase the fringe SNR (see Keen *et. al.*, MNRAS, 2001) (c) increasing the telescope aperture sizes.

To be somewhat quantitative about the impact of not meeting the various goals, the SNR for envelope tracking would be reduced by a factor of two (removing most of the safety margin) by *any one of*

- Increasing the read noise from 1.5e to 3.0e
- Decreasing the throughput from 20% to 12%
- Increasing the temporal and spatial wavefront errors from $\lambda/14$ to $\lambda/10$

Use of spatial filtering could *increase* the SNR by a factor of around 1.5.

The effect of degrading *all* of the goals to the bulleted values above would be to reduce the limiting magnitude to $H = 12.6$ and the number of potential targets to 24.

4.4.3 Variation with aperture size

The variation of predicted fringe tracking SNR with telescope aperture is shown in Figure 2. The uppermost curve assumes the detector read noise goal of 1.5e has been met. Further curves are given for read noise values of 2e (this has been achieved with a Rockwell PICNIC device and custom controller), 5e (Rockwell quotes this as standard for the AOMUX device), and 10e (Rockwell value for HAWAII device).

The number of potential targets as a function of telescope aperture is shown in Figure 3, for the same read noise values as Figure 2. A critical value for the SNR of 0.66 was assumed. The exponential function plotted in Figure 1 was used to interpolate between the quasar counts from the Veron catalogue.

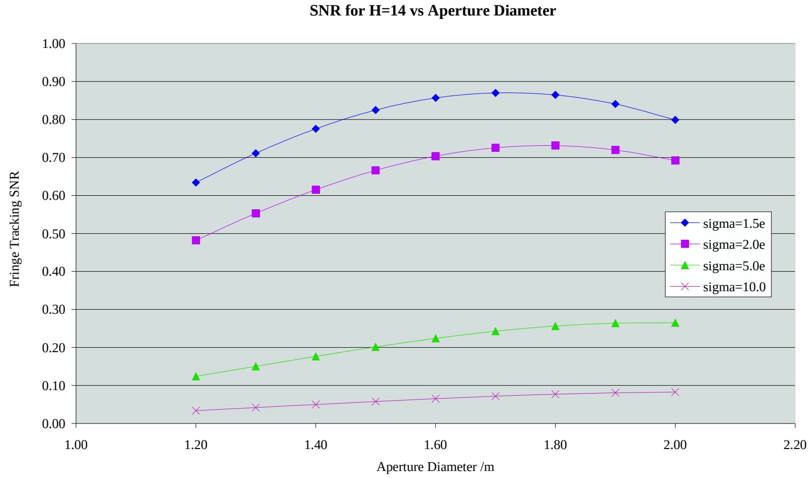

Figure 2: Variation of signal-to-noise ratio for fringe envelope tracking with aperture diameter.

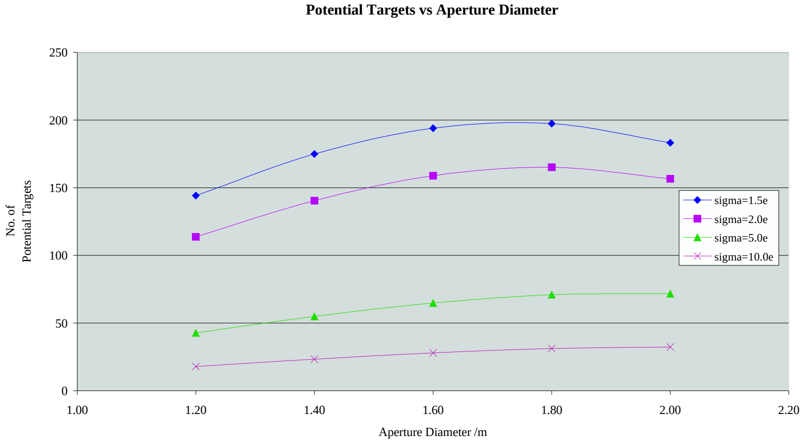

Figure 3: Number of potential targets as a function of aperture diameter.